The Mathematical Sciences in 2025

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The mathematical sciences are part of nearly all aspects of everyday life—the discipline has underpinned such beneficial modern capabilities as Internet search, medical imaging, computer animation, numerical weather predictions, and all types of digital communications. This report examines the current state of the mathematical sciences and explores the changes needed for the discipline to be in a strong position and able to maximize its contribution to the nation in 2025. It finds the vitality of the discipline excellent and that it contributes in expanding ways to most areas of science and engineering, as well as to the nation as a whole, and recommends that training for future generations of mathematical scientists should be re-assessed in light of the increasingly cross-disciplinary nature of the mathematical sciences. In addition, because of the valuable interplay between ideas and people from all parts of the mathematical sciences, the report emphasizes that universities and the government need to continue to invest in the full spectrum of the mathematical sciences in order for the whole enterprise to continue to flourish long-term.

Broadening of the Mathematical Sciences

Work in the mathematical sciences is increasingly integral to a growing array of research areas, including biology, medicine, social sciences, business, advanced design, climate, finance, and advanced materials. This work draws on the interplay of a wide range of techniques from mathematics, statistics, and computation and makes a critical contribution to economic growth, national competitiveness, and national security. As reliance on complex computer simulations and the analysis of large volumes of data increase, the discipline also naturally expands through its essential role in contributing the framework and tools required to perform such simulations and analyses. The ubiquity of computational simulations and exponential increases in the amount of data available for study are two major drivers of the increased "reach" of the mathematical sciences.

This broadening of the discipline should inform the nature and scale of funding of the enterprise, and education in the mathematical sciences should also reflect this evolution. The mathematical sciences have an exciting opportunity both to solidify their role in underpinning 21st century research and technology while maintaining the strength of the core, which is vital to the future of the mathematical sciences ecosystem. As explored in the full NRC report, the mathematical sciences community is achieving great success within this changing model. However, many mathematical scientists remain unaware of the expanding role for their field, which places a limitation in the community's ability to produce broadly trained students, and a community effort is needed to improve this situation.

Boundaries within the mathematical sciences, as well as boundaries between the mathematical sciences and the disciplines that use them, are blurring. Internal to the discipline, there is more collaboration and more research that melds insights from different subfields. This has enabled some of the most exciting recent advances in fields of study that were rarely brought together in the past. And in terms of external interactions, a sizeable amount of mathematical sciences research is actually conducted by, or with, people from outside of mathematical sciences departments. An inclusive view of the mathematical sciences and of the mathematical sciences community is called for.

In the face of the expanding role of the mathematical sciences and their consequent increase in impact, the adequacy of current federal funding for the discipline is a notable concern. The dramatic broadening of the role of the mathematical sciences over the past 15 years has not been matched by a comparable expansion in federal funding, either in the total amount or in the diversity of sources. The discipline, especially its core areas, is still heavily dependent on the National Science Foundation (NSF).

Other Trends Affecting the Mathematical Sciences

One significant change over the past 10-15 years has been an increase in the number of mathematical science institutes and their greater influence on the discipline and its community. These institutes now play an important role in helping mathematical scientists at various career stages to explore new areas of investigation, and they have facilitated new collaborations and helped to link the mathematical sciences with other fields. Collectively, these institutes have had an enormous impact in changing and broadening the culture of the mathematical sciences.

Another important trend is the rise of new modes of scholarly communication based on the Internet, making it easier for mathematical scientists to collaborate with researchers around the world. However, these new modes of collaboration, as well as any related impacts on publishing, will call for adjustments to the measures of quality control and professional accomplishments.

A final trend, which began decades ago and escalated in the 1990s, is the ubiquity of computing throughout science and engineering. Computation is often the means by which the mathematical sciences are applied in other fields, and it is the driver of many new applications in the discipline. More mathematical scientists need to have a richer understanding of computing as an intellectual discipline and a source of mathematical science challenges, and academic departments should assist in enabling this. A mechanism is also needed to ensure mathematical science researchers have access to computing power at an appropriate scale.

Adjusting the Educational Path in the Mathematical Sciences

The expansion of research opportunities in the mathematical sciences—and in careers that build on mathematical sciences education, regardless of their academic specialty—necessitates changes in the way that students are educated. It also calls for an effective plan to attract a greater number of talented young people into the mathematical sciences. The existing demand for people with strong mathematical science skills is likely to grow as positions requiring these skills continue to expand.

This growing demand has educational implications for the mathematical sciences community as it prepares students for a broad range of science, technology, engineering, and mathematics (STEM) careers. Specifically, it is essential that mathematical science educators at the K-12 and undergraduate levels convey to students how mathematical science coursework is used and the careers it can lead to. Graduate students also should learn this information so they can pass it along as faculty members to their students.

Mathematical sciences curricula need attention. The educational offerings of typical departments in the mathematical sciences have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large. This diversification entails a need for new courses, new majors, new programs, and new educational partnerships with those in other disciplines, both inside and outside universities. New educational pathways for training in the mathematical

sciences need to be created—for students in mathematical sciences departments, for those pursuing degrees in science, medicine, engineering, business, and social science, and for those already in the workforce needing additional quantitative skills. There are exciting opportunities for reaching out to new cadres of students.

While the mathematical sciences enterprise has tremendous responsibilities for educating students across the range of STEM fields, it is of course also essential to replenish itself. The federal government should establish a national program to provide extended enrichment opportunities for pre-collegiate students with an unusual talent in the mathematical sciences.

Mathematics and statistics departments, in concert with their university administrations, also should carefully consider the different types of students they are attracting, and wish to attract, and identify the top priorities for educating these students. This should be implemented for bachelor's, master's, and Ph.D.-level curricula. Every academic department in the mathematical sciences should explicitly incorporate recruitment and retention of women and underrepresented groups into the responsibilities of the faculty members in charge of the undergraduate and graduate programs, and in faculty hiring and promotion. With access to the proper resources, departments could adapt successful recruiting and mentoring programs that have been pioneered at a number of schools. These resources could also help locate and correct any disincentives that may exist in the department.

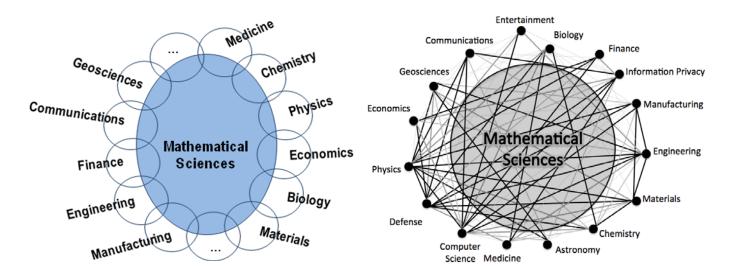
There is another consideration for strengthening the mathematical sciences enterprise: The market for mathematical sciences talent is now global, and the United States is in danger of losing its global preeminence in the discipline. The policy of encouraging the growth of the U.S.-born mathematical sciences talent pool should continue, but it needs to be supplemented by programs to attract and retain mathematical scientists from around the world, who increasingly have options in their own countries.

More professional mathematical scientists should become involved in explaining the nature of the mathematical sciences enterprise and its extraordinary impact on society, and academic departments should find ways to reward such work. Professional societies should expand existing efforts and work with funding entities to create an organizational structure with the goal of publicizing advances in the mathematical sciences.

Stresses on the Horizon

Financial stresses on students and their families, reductions in government funding for universities, and new educational technologies are creating pressure to lower costs. Mathematical science departments, particularly those in large state universities, have a tradition of teaching service courses for nonmajors this approach helps to fund positions for mathematical scientists at all levels, particularly junior faculty and graduate teaching assistants. The desire to reduce expenses is pushing students to take some of their lowerdivision studies at state and community colleges or online. It is also leading university administrations to hire a category of faculty members who have greater teaching loads, reduced expectations of research productivity, and lower salaries. Another approach has been to implement online courses that can be taught with less ongoing faculty involvement. The recent emergence of massive online courses, some with mathematical science content, is another potential disruption to the status quo.

Because of their important role in teaching service courses, the mathematical sciences will be disproportionately affected by these changes. However, there may also be expanded opportunities to train students from other disciplines or those who are already in the workforce. Academic departments in mathematics and statistics should begin the process of rethinking and adapting their programs to keep pace with the evolving academic environment and to be sure to engage actively so that they have a seat at the table when curricula are modified and online courses with mathematical sciences content are created. The professional societies have important roles to play in mobilizing the community in these matters as well—that is, through mechanisms such as opinion articles, online discussion groups, policy monitoring, and conferences. Mathematical scientists should work proactively—through funding agencies, university administrations, professional societies, and within their departments—to be ready for upcoming changes to the college and university environment.



Caption: The mathematical sciences and their interfaces in 1998 (left) and 2013 (right). In the figure on the left, the empty "bubbles" are meant to reflect the many other intersections that are not explicitly labeled. As the number of interfaces increase, the mathematical sciences themselves broaden in response and play an important role in a highly-integrated system. These schematics are notional, based on the committee's varied and subjective experience rather than on specific data.

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