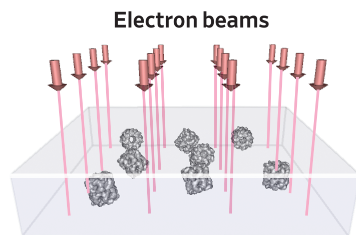
Mathematics of Imaging: Seeing by Solving

We often depend on images to make sense of our surroundings, be it through explorations of complex molecules that underpin scientific advances, examinations of structures inside the human body to help treat illnesses, or glimpses into the outer reaches of our universe. However, most images begin as a collection of data and must be converted to something that can be interpreted by the human eye. Mathematical and statistical techniques make this possible.

SEEING SMALL

Researchers can make three-dimensional images of proteins smaller than a nanometer, thanks to a technique called cryogenic electron microscopy.

In this process, mathematics is used to reduce noise and create an interpretable image from the data.



DATA COLLECTION

First, a large sample of identical molecules is flash frozen and an array of electron beams is directed through the sample.

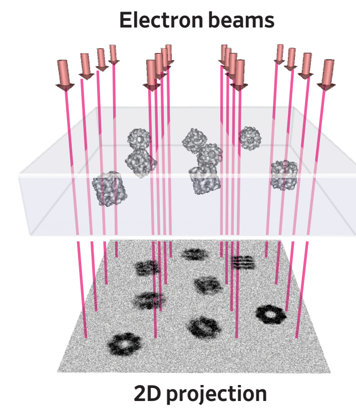
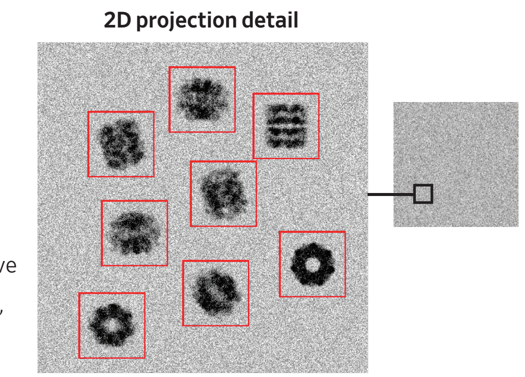


IMAGE RECONSTRUCTION

The electron beams generate two-dimensional projections of a molecule.

In order to construct a three-dimensional image of the molecule, researchers need to solve a problem known as "angular reconstruction."



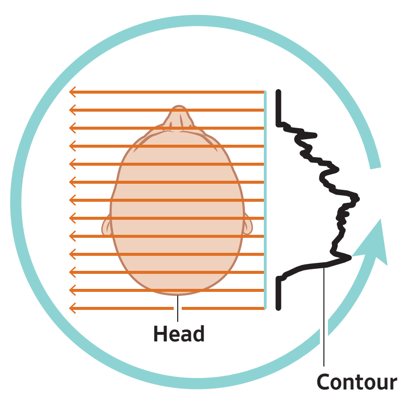
FINAL IMAGES

These projections are tiny and hard to distinguish from noise, requiring specialized mathematical processing techniques. The Fourier transform aids in approximating the solution.

SEEING WITHIN

Doctors can view bones and organs in the body through computed tomography, often referred to as a CT scan.

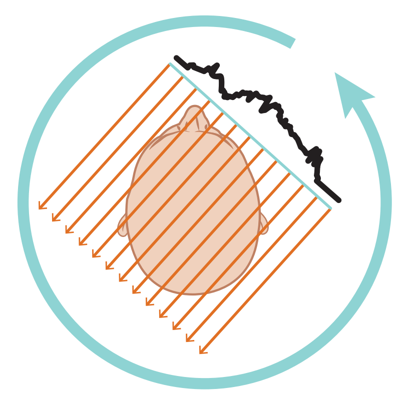
These scans use mathematical tools, such as the Radon transform and filtered back projection, to piece together X rays taken from many angles and create a coherent image of the body part being scanned.



In a CT scan, X rays are passed through the body from many different angles toward a set of detectors that measure the intensity of the rays.

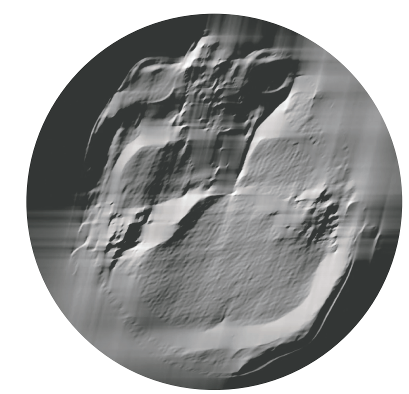
The data are captured as a series of contours.

This collection is called a Radon transform.



Each contour provides a glimpse into what the scanned part looks like from a different angle.

But the views are often incomplete or unclear.



Filtered back projection—one approach for solving this inverse problem—involves filtering and distributing the contours into an image.

SEEING BEYOND

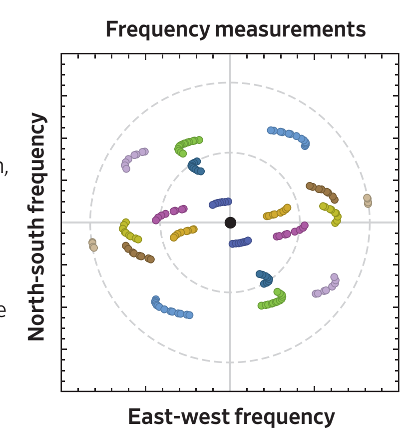
With the Event Horizon Telescope, scientists recently captured the first image of a black hole, approximately 55 million light-years away from Earth.

This monumental discovery was enabled by mathematical tools, including the Fourier transform, imaging algorithms, and optimization.

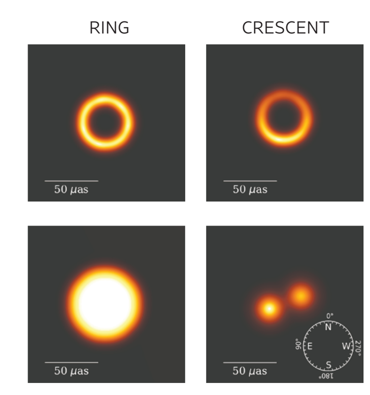
To visualize the black hole, each pair of **telescopes** collects a frequency data point of the incoming radiation, measured relative to their positions.

The time delays at these different locations are exploited to correct noise in these signals.

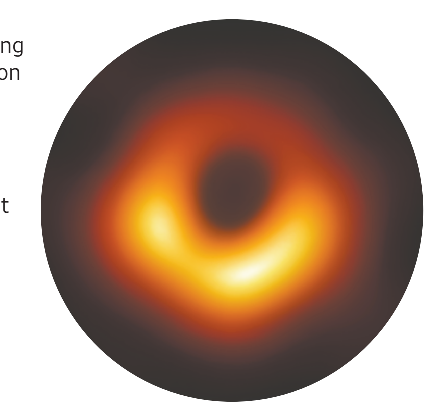
Earth's rotation allows scientists to capture more measure- ments from this small number of telescopes.



The data are sparse and noisy, and many possible images might fit this data. Researchers use mathematical imaging algorithms to produce images from the incomplete information about the frequency measurements.



Machine learning and optimization techniques are involved in producing and validating the image that best fits the data.



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Sources: Images by Grigore Pintilie, Stanford University (Seeing Small); Samuli Siltanen, University of Helsinki (Seeing Within); and Katherine L. Bouman, California Institute of Technology (Seeing Beyond). Molecule (Seeing Small) by H.-T. Tran, J. Lee, H. Park, J.-G. Kim, S. Kim, Y.-J. Ahn, and L.-W. Kang, 2019. Crystal structure of Chaperonin *GroEL* from *Xanthomonas oryzae* pv. *oryzae*, *Crystals* 9(8):399, https://doi.org/10.3390/cryst9080399. CT scan used with permission by Andrew Ciscel (Seeing Within).